How to quantum shuffle cards – mixing time and cutoff profiles

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joint work with Amaury Freslon (Orsay) & Lucas Teyssier (Vienna)

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Poincaré (1912): when we are playing cards, after a sufficiently long time, all the permutations of cards appear with equal probabilities.

CHAPITRE XVI.

QUESTIONS DIVERSES.

225. Battage des cartes. — Je me suis occupé dans l'introduction des problèmes relatifs au joueur qui bat un jeu de cartes. Pourquoi, quand le jeu a été battu assez longtemps, admettons-nous que toutes les permutations des cartes, c'està-dire tous les ordres dans lesquels ces cartes peuvent être rangées, doivent être également probables? C'est ce que nous allons examiner de plus près.

→ "random walk theory"

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A "LAZY" CARD SHUFFLE BY RANDOM TRANSPOSITIONS

- Spread the cards on a table;
- Select one card uniformly at random, then put it back;
- Select a second card in the same way;
- Swap the two cards if different;
- Otherwise, do nothing.

Interpretation:

- $\mu_{\text{tran}} =$ uniform measure on transpositions in the permutation group S_N
- $\mu_N = \frac{N-1}{N}\mu_{\text{tran}} + \frac{1}{N}\delta_{\text{id}}$
- Random walk on S_N driven by μ_N : the distribution at the *k*-th step is

$$\mu_N^{*k}(\sigma) \coloneqq \sum_{\substack{\sigma_1, \dots, \sigma_k \in S_N \\ \sigma_1 \cdots \sigma_k = \sigma}} \mu_N(\sigma_1) \cdots \mu_N(\sigma_k), \quad \sigma \in S_N.$$

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A "LAZY" CARD SHUFFLE BY RANDOM TRANSPOSITIONS

Question Does this acutally mix the cards ? **Answer** Yes. μ_N^{*k} converges weakly to the Haar measure on S_N .

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A "LAZY" CARD SHUFFLE BY RANDOM TRANSPOSITIONS

Question Does this acutally mix the cards ?

Answer Yes. μ_N^{*k} converges weakly to the Haar measure on S_N .

Question How and when?

Answer Diaconis-Shahshahani 81':

- Before $N \ln(N)/2$ steps, the distribution stays far from uniform;
- After $N \ln(N)/2$ steps, the distribution suddenly drops to uniform.



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CUTOFF PHENOMENON FOR RANDOM TRANSPOSITIONS

Denote μ_{Haar} = Haar measure on S_N . The total variation distance

$$d(\mu_N^{*k}, \mu_{\text{Haar}}) \coloneqq \sup_{A \subset S_N} |\mu_N^{*k}(A) - \mu_{\text{Haar}}(A)| = \frac{1}{2} \|\mu_N^{*k} - \mu_{\text{Haar}}\|_1$$

Theorem (Diaconis-Shahshahani 81')

For $\epsilon > 0$ *, as* $N \to \infty$

$$d(\mu_N^{*\lfloor(1-\epsilon)N\ln(N)/2\rfloor}, \mu_{\text{Haar}}) \to 1, \quad d(\mu_N^{*\lfloor(1+\epsilon)N\ln(N)/2\rfloor}, \mu_{\text{Haar}}) \to 0.$$

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Theorem (Diaconis-Shahshahani 81')

For $\epsilon > 0$, as $N \to \infty$



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How does the "cutoff" occur in the short window?

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How does the "cutoff" occur in the short window?

Theorem (Teyssier, Ann. Proba. 20')

For $c \in \mathbb{R}$ *and* $N \to \infty$ *,*

$$d(\mu_N^{*\frac{1}{2}(N\ln(N)+cN)},\mu_{\text{Haar}}) \to d(\text{Poiss}(1+e^{-c}),\text{Poiss}(1)),$$

where $Poiss(\lambda) = Poisson$ law of parameter λ .

"cutoff profile" Ouantum cutoff

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 = collections of "observables" in physics
- (locally compact) topological space $\Omega \leftrightarrow$ commutative C*-algebra $C_0(\Omega) \subset B(\ell_2(\Omega))$ "quantum topological space" \leftrightarrow noncommutative C*-algebra
- symmetries on topological spaces ↔ topological groups
 "quantum symmetries on classical/quantum topological spaces"
 ↔ topological quantum groups

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And μ_{A} by the formation of the translate of h_{A} by an the spectrum state M_{A} and M_{A} by the translation of the state	And $r_{k,k}$ is the investigation of the total and $r_{k,k}$ is the total constraints of the t		てんれん ベームボロ ウィン・ハース・ハー	
The proof is calculated with the three strength of the streng	The node restriction of the set	where $\lambda_{\rm ch},\lambda_{\rm S}$ is corrections.	ing to the commutant of the baseable and the	$\hat{r}_{\beta},\vartheta y$ are the respective
Is drawn as the start has a start has a start has a start of the start has a start has a start of the start has a start	be the one action of a soft shortened burd short has been as the soft shortened burd short shortened burd shortened burdened bur	The proof is str	ightforward, given Theorem 1 and the above-	competations.
$\label{eq:second} \begin{split} & L_{k} Loss the distribution of the $	$\label{eq:second} \begin{array}{l} L_{s} \mbox{ Loss } \mbox{ tot } \mbox{ Loss } $	Let us now not not used 17-dime	that, at the classical level which we have as sional subspace given by the following restric	et left as far, there is a tion:
The comparison $(k_1,k_2,k_3,k_4,k_4,k_4,k_4,k_4,k_4,k_4,k_4,k_4,k_4$	The comparison behavior as the first set of the set of		J _A J _A belong to the algebra processed by A	and B.
$\begin{split} & (52) & n_{c} \geq d^{-}_{2} P(L_{c} + 0) \\ & (52) & m_{c} = n_{c} \left(\frac{1}{1 r^{-} r^{-} r^{-} r^{-} r^{-}} \right) \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r} \left(\frac{n_{c}}{r^{-} r^{-} r^{-}} \right)^{-} \\ & (52) & m_{c} = n_{c} \left(\frac{1}{1 r^{-} r^{-} r^{-} r^{-}} \right) \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r} \left(\frac{n_{c}}{r^{-} r^{-} r^{-}} \right)^{-} \\ & (52) & \frac{1}{r^{-} r^{-} r^{r$	$\begin{split} & (52) \qquad n_{1} \geq d_{2} = N(\lambda_{1} + 0, \\ & (2) = n_{2} = n_{1} + n_{2} + 1(\lambda_{1} + 1), \\ & (1 + \lambda_{1} + 1), \\ &$	The correspond large top man a	ng classical relation con only have a hearing in i.e., neglecting the other levision means, or	e value. In the limit of ne gets the following:
$\begin{split} & (31) \qquad \qquad m_{0}=m_{0}\left(\frac{1}{m_{0}^{2}+m_{0}^{2}-m_{1}^{2}}\right)^{3}, \frac{1}{2}+\frac{1}{2}+\frac{1}{m_{0}^{2}}\left(\frac{m_{0}}{m_{0}}\right)^{2}, \\ & 1 \leq 1$	$\begin{split} & (23) \qquad n_A = m \left(t^2 + b_A - t^2 + b_B - t^2 + t^2 + b_B - t^2 + b_B $	(6.29)	$m_{i} \ge \sqrt{2}m_{W}$ if $\lambda_{A} > 0$,	
Here a the task is between the two supersides of the spectra $\Lambda_{\rm p}$ or fluct the value -1 is the next state halfs by two $-\gamma$. So, for a the substate time is a physical -1 is the sect state -1 is the section -1 is th	Here a the task is between the two sequencing of the opticator λ_{ij} , we find the sequence λ_{ij} is the sequence of the sequence λ_{ij} and the sequence λ_{ij} is the sequence of the sequence λ_{ij} and the sequence λ_{ij} are sequence the sequence λ_{ij} and the sequence λ_{ij} and the sequence λ_{ij} are sequence the sequence λ_{ij} and the sequence λ_{ij} are sequence λ_{ij} and the sequence λ_{ij} are sequence the sequence λ_{ij} and the sequence λ_{ij} are sequence λ_{ij} are sequence the sequence λ_{ij} are sequence λ_{ij} are sequence the sequence λ_{ij} are sequen	(6.25)	$u_A = u_1 \left(3 \frac{x^2 + 8x + 14}{x^2 + 8x + 15}\right)^{51}, \frac{1}{4}x + \frac{5}{4} = \left(\frac{1}{2}\right)^{51}$	<u>⇒</u>)'.
In order to bundless our interpretation of the standard model into a perdictive theory is down and the down and the down and here the end of the standard star is the start of the third space F . If the start is the start of the down in the down and the down and the start of the down of the down is the start of the down of the down is the start of the down of the down is the start of the down of th	In order to boundoms use interpretation of the standard model into a predictive theory is a bound of the standard model in the standard model in the standard model in the standard model is a standard field field model applies of the field space $F_{\rm c}$ is a standard field model model model model model model model model in the standard model model is a standard model of 1.5 stans into induction of model	Here x is the no x = 1 is the non- prediction, but would show the above flavores i	in between the two eigenvalues of the operation t natural, insiding to $m_1 = 2m_2$. None of thes if they were usually satisfied by the physical z the change of parameterization in the stand- capalitatively a good one at the channel level	or Λ_{A} , so that the value so relations is a physical values of m_{0} and m_{0} it and model given by the n.
1) Find a nontrivial finite quantum group of exametrize of the finite space F . 2) Determine the effectuar of the Chiller inspire of the finite space F , group by linear map from the space HoR of 1 forms into the algebra of endomerphisms of R_F which are a first The α .	I Find a matrixed finite quantum group of symmetries of the finite space F . 2) Determine the structure of the United algebra of the lastic space F , given by finite map from the space Field of 1 forms into the algebra of endomerphisms of R_F which to a 1 form $\sum n_i$ dip associates $\sum n_i (D, h_i)$.	In order to train it is isometical.)	form our interpretation of the standard model a solar the following availables:	iata a produtive theory
2) Determine the effectuate of the United algebra of the hinto space F , given by the linear map from the space HOR of I forms into the algebra of endomorphisms of R_F which we determine the space $R_{\rm eff}$ of R_F	2) Determine the effecture of the Chiller i algebra of the limits space F_1 given by linear map from the space Holl of 1 forms into the algebra of endomerphisms of R_F which to a 1 form $\sum n_i$ di ₁ amoviates $\sum n_i D,h_i $.	1) Find a nests		e faibe space F.
linear map from the space HOR of 1-forms into the algebra of endomorphisms of H ₂ -	linear map from the space HOH of 1-forms into the algebra of endomorphisms of Hy which to a 1-form $\sum n_i$ ofly associates $\sum n_i D,h $.	2) Determine 11	e state base of the Chiller's algebra of the high	o space P , group by sur-
	which to a 1-form $\sum n_i$ dly associates $\sum n_i[D,h]$.	lines map from	the space HOH of 1-forms into the algebra of	d endomorphisms of Hy-

"... it is important to solve the following problems: Find a nontrivial finite quantum group of symmetries of the finite space F."

Alain Connes

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• A (classical) permutation matrix $C = [c_{ij}]_{1 \le i,j \le N} \in S_N$ is such that

$$c_{ij} \in \{0,1\}, \quad CC^t = C^t C = I.$$

The algebra $C(S_N)$ of functions on S_N is generated by the functions $C \mapsto c_{ij}$.

Quantum permutations (Shuzhou Wang): Consider the universal C*-algebra A generated by operators (u_{ij})_{1≤i,j≤N} s.t. for the matrix U = [u_{ij}]_{1≤i,j≤N},

$$u_{ij} = u_{ij}^* = u_{ij}^2, \quad UU^t = U^t U = I.$$

Intuitive notation: quantum permutation group $S_N^+ = (A, U)$ and $A = {}^{"}C(S_N^+)"$.



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Intuitive notation: quantum permutation group $S_N^+ = (A, U)$ and $A = C(S_N^+)''$.

• Interpretation in language of quantum physics: u_{ij} 's = observables measurement on a quantum state $\xi \rightarrow$ random permutation

$$\mathbb{P}(i \to j) = \langle \xi | u_{ij} | \xi \rangle$$

Atserias,Lupini,Mancinska,Roberson,..., 19'-20': quantum permutations much better than the classicals one when contructing strategies in non-local games on graphs.

• Analogue of group multiplications: *-homomorphism

$$\Delta: C(S_N^+) \to C(S_N^+) \otimes C(S_N^+), \quad u_{ij} \mapsto \sum_{k=1}^N u_{ik} \otimes u_{kj}.$$

• Analogue of convolutions: for two states $\varphi_1, \varphi_2 \in C(S_N^+)^*$,

$$\varphi_1 * \varphi_2 \coloneqq (\varphi_1 \otimes \varphi_2) \circ \Delta.$$

- Analogue of Haar measure: \exists unique state $h \in C(S_N^+)^*$ s.t. for all state $\varphi \in C(S_N^+)^*$, $\varphi * h = h * \varphi = h$, called the Haar state.
- Analogue of total variation distance: the distance in $C(S_N^+)^*$, for two states φ_1, φ_2 ,

$$d(\varphi_1, \varphi_2) \coloneqq \frac{1}{2} \|\varphi_1 - \varphi_2\|_{C(S_N^+)^*} \big(= \sup_{p = p^* = p^2 \in C(S_N^+)^{**}} |\varphi_1(p) - \varphi_2(p)| \big).$$

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QUANTUM RANDOM TRANSPOSITIONS

Recall: classical random transpositions given by $\mu_N = \frac{N-1}{N}\mu_{\text{tran}} + \frac{1}{N}\delta_{\text{id}}$

• μ_{tran} is unif distribution on $C := \{ \text{transpositions} \}$. Note that *C* is a conjugacy class, so for $\mathbb{E} = |S_N|^{-1} \int \operatorname{ad}(\sigma) d\sigma$,

$$\int_{S_N} f d\mu_{\text{tran}} = \int_{S_N} (\mathbb{E}f) d\mu_{\text{tran}} \quad (= (\mathbb{E}f)((12))).$$

• there is a similar conditional expectation \mathbb{E} from $C(S_N^+)$ onto adjoint-invariant elements. We consider analogously

$$\varphi_{\operatorname{tran}}(f) = (\pi \circ \mathbb{E}f)((12)), \quad f \in C(S_N^+),$$

where $\pi : C(S_N^+) \to C(S_N)$ denotes the abelianization. (Intuitively unif distribution on the quantum conjugacy class of transpositions)

- counit $\varepsilon : C(S_N^+) \to \mathbb{C}$, unique state s.t. $\varepsilon * \varphi = \varphi, \forall \varphi \in C(S_N^+)^*$.
- Problem: cutoff for $\varphi_N \coloneqq \frac{N-1}{N}\varphi_{\text{tran}} + \frac{1}{N}\varepsilon$?

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CUTOFF FOR QUANTUM RANDOM TRANSPOSITIONS

$$\varphi_N : C(S_N^+) \to \mathbb{C}, \quad \varphi_N = \frac{N-1}{N} \varphi_{\text{tran}} + \frac{1}{N} \varepsilon$$

Theorem (Freslon-Teyssier-W, PTRF 22')

For $\epsilon > 0$ *, as* $N \to \infty$ *,*

$$d(\varphi_N^{*\lfloor (1-\epsilon)\frac{N\ln(N)}{2}\rfloor},h)\to 1, \quad d(\varphi_N^{*\lfloor (1+\epsilon)\frac{N\ln(N)}{2}\rfloor},h)\to 0.$$

Moreover we have the cutoff profile: for $c \in \mathbb{R}$ *, as* $N \to \infty$ *,*

$$d(\varphi_N^{*\lfloor \frac{1}{2}(N\ln(N)+cN)\rfloor}, h)$$

$$\rightarrow d\left(D_{\sqrt{1+e^{-c}}}\left(\operatorname{Meix}^+\left(\frac{1-e^{-c}}{\sqrt{1+e^{-c}}}, \frac{-e^{-c}}{1+e^{-c}}\right)\right) * \delta_{e^{-c}}, \operatorname{Meix}^+(1,0)\right)$$

where: - $D_r(\mu)$ the r-dilation of μ (i.e. $rX \sim D_r(\mu)$ if $X \sim \mu$) - Meix⁺ denotes the free Meixner law. Free Meixner laws are introduced by Bozejko, Bryc, Saitoh, Yoshida, as analogues of classical Meixner laws. For $a \in \mathbb{R}, b \ge 1$,

$$d \operatorname{Meix}^+(a,b)(t) = rac{\sqrt{4(1+b)-(t-a)^2}}{2\pi(bt^2+at+1)}dt + \operatorname{atoms.}$$

• b = 0: free Poisson law (i.e. Marchenko-Pastur law) for $\lambda > 1$

$$d \operatorname{Poiss}^+(\lambda, \alpha)(t) = \frac{1}{2\pi\alpha t} \sqrt{4\lambda\alpha^2 - (t - \alpha(1 + \lambda))^2} dt$$

• a = b = 0: free semicircular law $(2\pi)^{-1}\sqrt{4 - t^2}dt$.

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STRATEGY OF THE PROOF I: $\varphi_{\mathrm{tran}}^{*k}$, Case c > 0

$$\varphi_N : C(S_N^+) \to \mathbb{C}, \quad \varphi_N = \frac{N-1}{N} \varphi_{\text{tran}} + \frac{1}{N} \varepsilon$$

- Recall that we aim to understand $d(\varphi_N^{*\lfloor \frac{1}{2}(N\ln(N)+cN)\rfloor}, h)$.
- Consider first $d(\varphi_{\text{tran}}^{*\lfloor\frac{1}{2}(N\ln(N)+cN)\rfloor}, h)$

• If c > 0, then $\varphi_{\text{tran}}^{*\lfloor \frac{1}{2}(N \ln(N) + cN) \rfloor} \in L^1(S_N^+)$ "absolutely continuous".

$$\begin{split} d(\varphi_{\text{tran}}^{*\lfloor\frac{1}{2}(N\ln(N)+cN)\rfloor},h) &= \frac{1}{2} \|\varphi_{\text{tran}}^{*\lfloor\frac{1}{2}(N\ln(N)+cN)\rfloor} - h\|_{L^{1}(S_{N}^{+})} \\ &\leq \frac{1}{2} \|\varphi_{\text{tran}}^{*\lfloor\frac{1}{2}(N\ln(N)+cN)\rfloor} - h\|_{L^{2}(S_{N}^{+})} \end{split}$$

computable via Fourier analysis and Chebychev polynomials.

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STRATEGY OF THE PROOF II: $\varphi_{\text{tran}}^{*k}$, Case c < 0

• If c < 0, then $\varphi_{\text{tran}}^{*\lfloor \frac{1}{2}(N \ln(N) + cN) \rfloor} \notin L^1(S_N^+)$ non absolutely continuous

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STRATEGY OF THE PROOF II: $\varphi_{\text{tran}}^{*k}$, Case c < 0

- If c < 0, then $\varphi_{\text{tran}}^{*\lfloor \frac{1}{2}(N \ln(N) + cN) \rfloor} \notin L^1(S_N^+)$ non absolutely continuous
- It suffices to consider $C(S_N^+)_{\text{central}} = \mathbb{C}^*$ -subalg generated by $\sum_i u_{ii}$.

$$d(\varphi_{\text{tran}}^{*\lfloor\frac{1}{2}(N\ln(N)+cN)\rfloor},h) = \|(\varphi_{\text{tran}}^{*\lfloor\frac{1}{2}(N\ln(N)+cN)\rfloor}-h)|_{C(S_N^+)_{\text{central}}}\|_{C(S_N^+)_{\text{central}}^*}$$

• $C(S_N^+)_{\text{central}} \simeq C([0, N]) \longrightarrow \text{ a classical measure}$

$$\int_0^N f dm_k^{(N)} = \varphi_{\text{tran}}^{*k}(f).$$

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Proposition (Freslon-Teyssier-W)

$$m_k^{(N)} = \alpha_N(k)\delta_{\tilde{N}(k)} + \tilde{m}_k^{(N)},$$

where $\alpha_N(k) \in \mathbb{R}$ and $\tilde{N}(k) \notin [0, 4]$, and $\tilde{m}_k^{(N)} \in L^2([0, 4], \text{Poiss}^+(1, 1))$.

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The previous methods lead to the cutoff for φ^{*k}_{tran}.
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- The previous methods lead to the cutoff for φ^{*k}_{tran}.
 (Contrary to the classical setting; purely quantum!)
- How to pass to φ_N^{*k} ? $\varphi_N^{*k} \notin L^1(S_N^+)$ never absolutely continuous!

$$\varphi_N = \frac{N-1}{N}\varphi_{\rm tran} + \frac{1}{N}\varepsilon$$

The previous Fourier analytic tools break down.

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- Idea: φ_N^{*k} = randomized $\varphi_{\text{tran}}^{*k}$
 - Flip a biased coin with probability 1/N for heads ;
 - Tail \rightsquigarrow pick φ_{tran} ; head \rightsquigarrow do nothing. $X_k \sim \text{Binom}(k, \frac{N-1}{N})$
 - $\varphi_N^{*k} = \mathbb{E}(\varphi_{\text{tran}}^{*X_k})$

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- The previous methods lead to the cutoff for $\varphi_{\text{tran}}^{*k}$. (Contrary to the classical setting; purely quantum!)
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 - $\varphi_N^{*k} = \mathbb{E}(\varphi_{\text{tran}}^{*X_k})$

Proposition (Freslon-Teyssier-W)

For $c \in \mathbb{R}$, $\|\varphi_N^{*\lfloor \frac{1}{2}(N\ln(N)+cN)\rfloor} - \varphi_{\text{tran}}^{*\lfloor \frac{1}{2}(N\ln(N)+cN)\rfloor}\|_{C(S_N^+)^*} \to 0, \quad as \ N \to \infty.$

Simeng Wang (Harbin Institute of Technology)

Thank you very much!

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